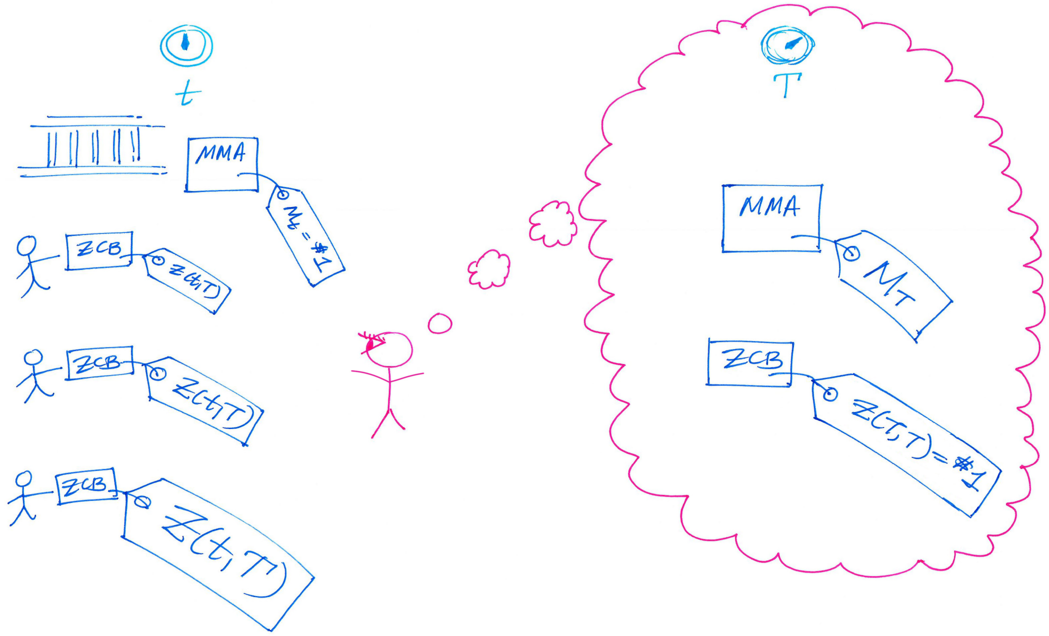
Constancy of asset price ratios (for deterministic final prices)

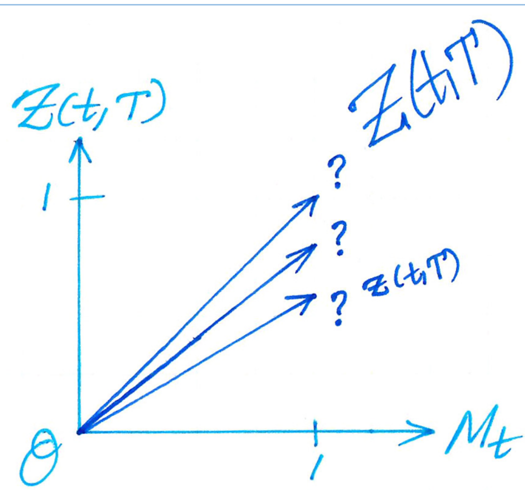
Si

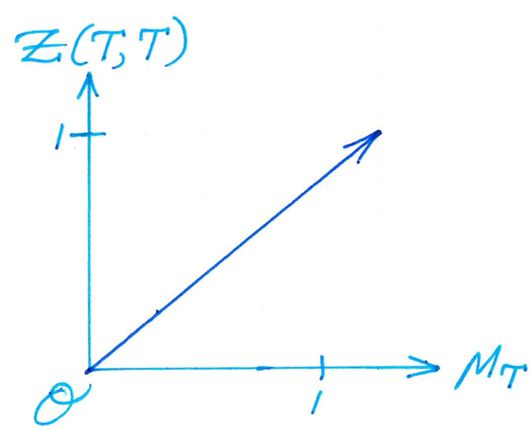
Present time t	Future time T



At present time t, a financial modeler observes the common price of money market accounts and a diversity of prices for zero coupon bonds. At present time t, the financial modeler is also sure of the future price of the money market account at future time T and the common price of all ZCBs maturing at T.

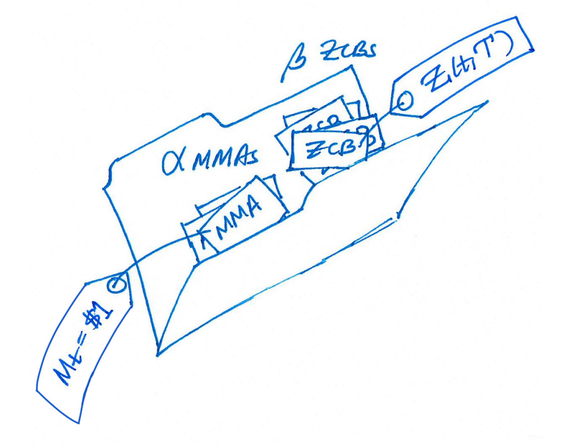
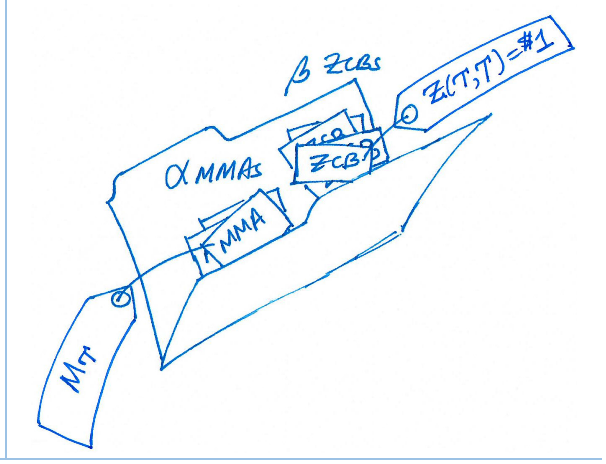
Qu





Can the we determine a relationship among asset prices describing the price at which ZCBs that mature at future time T will for the most part trade at present time t?

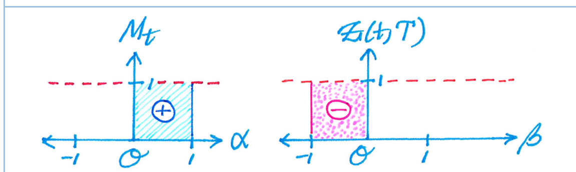
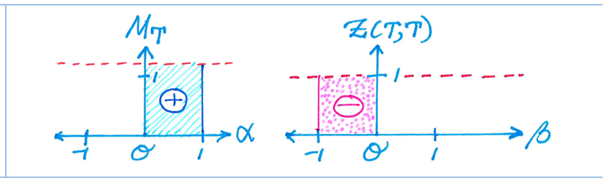
Constancy of asset price ratios (for deterministic final prices)

Si	Present time t	Future time T
		

A portfolio contains stacks of assets. Mechanisms exist to allow the number of units of an asset in a stack to be negative.

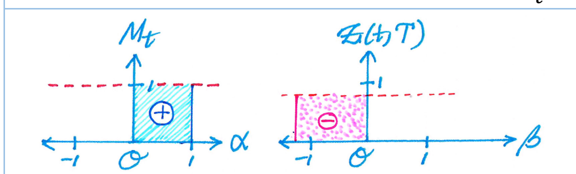
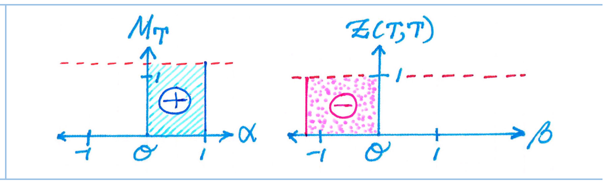
Qu

(I)

Present time t	Future time T
	

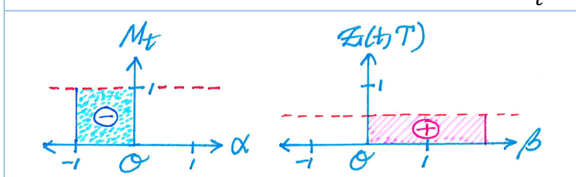
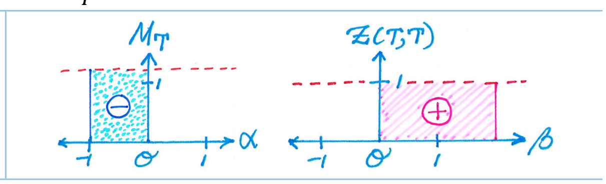
$$\frac{Z(t,T)}{M_t} > \frac{Z(T,T)}{M_T}$$

(II)

Present time t	Future time T
	

$$\frac{Z(t,T)}{M_t} = \frac{Z(T,T)}{M_T}$$

(III)

Present time t	Future time T
	

$$\frac{Z(t,T)}{M_t} < \frac{Z(T,T)}{M_T}$$

The price of a stack of assets equals the product of the number of units of the asset in the stack and the price of each unit of the asset in the stack. The price of a portfolio equals the price of each asset stack in the portfolio.

E	$V(t) = \alpha Z(t,T) + \beta M_t$	$V(T) = \alpha Z(T,T) + \beta M_T$

Constancy of asset price ratios (for deterministic final prices)

Fundamental assumptions about agent behavior: There exist **arbitrageurs** who look for opportunities to create portfolios of assets that have a price of $\$0$ at present time t and, despite no additional financing, then have positive price at future time T. Such a portfolio is a special case of what will be more broadly defined later as an **arbitrage portfolio**.

Any arbitrage portfolio that can be created by available asset prices will be almost immediately created by arbitrageurs. The availability of sellers and buyers willing to sell and buy assets at the prices needed for the creation of the arbitrage portfolio will quickly dwindle until the arbitrage portfolio is no longer possible to create. When an opportunity to construct arbitrage portfolios arises, the opportunity is short-lived, and the number of units of arbitrage portfolios that can be created during the opportunity is limited.

Only portfolios that are not arbitrage portfolios are expected to be observed to be traded in any significant extent.

No arbitrage postulate: There are no arbitrage portfolios.

N

Figure rows (I) and (III) above show that arbitrage portfolios can be constructed when $\frac{Z(t,T)}{M_t} \neq \frac{Z(T,T)}{M_T}$, so the no-arbitrage postulate requires that

$$\frac{Z(t,T)}{M_t} = \frac{Z(T,T)}{M_T}$$

Constancy of asset price ratios for deterministic final prices

Hypothesis	*Conclusion*
• There are no arbitrage portfolios • Asset A has price $A(t)$ at time t • Asset N has price N_t at time t • N_t and N_T are both > 0 • $A(T)$ and N_T are both known at time t	$\frac{A(t)}{N_t} = \frac{A(T)}{N_T}$

The slope of the secant line from the origin to the point $(N_t, A(t))$ representing an instantaneous pair of prices is constant.

Qu